

Light Mesons from Heavy B and Hyperon Decays

Patrick J. O'Donnell^{a*}

^aDepartment of Physics,
University of Toronto,
Toronto, Ont. Canada. M5S 1A7

Decays of heavy mesons and of heavy hyperons are used to provide tests of the standard model and information about new mixing schemes for the η and η' mesons. These include the two body decays $B_s \rightarrow J/\psi M$ and $B_d \rightarrow J/\psi M$, $B \rightarrow \eta(\eta')K(K^*)$ and $\Lambda_b \rightarrow \Lambda\eta(\eta')$, semileptonic D decays, and properties of radially excited mesons.

1. Introduction

In this talk, I consider the two body decays of heavy mesons and of heavy hyperons $B_s \rightarrow J/\psi M$ and $B_d \rightarrow J/\psi M$, $B \rightarrow \eta(\eta')K(K^*)$, $\Lambda_b \rightarrow \Lambda\eta(\eta')$ and the semileptonic D decays. Tests of the standard $\eta - \eta'$ mixing and of properties of radially excited mesons are given.

2. Relations for two body $B \rightarrow J/\psi$ Decays and tests for $\eta - \eta'$ mixing.

Nonleptonic two-body decays $B^0 \rightarrow J/\psi M$ and $B_s \rightarrow J/\psi M$ are of particular interest since one of these is the “golden channel” - $J/\psi K_S$, important for CP violation. In the inactive spectator approach the analysis of large groups of different decays related by symmetries is simplified since the bound $c\bar{c}$ pair is a singlet under color, isospin and flavor SU(3) and is an eigenstate of C and P . The two-meson final state has a unique color coupling so that there are **selection rules** for all decays without the spectator quark in the final state:

$$A[B^0 \rightarrow J/\psi M(\bar{q}s)] = 0, A[B_s \rightarrow J/\psi M(\bar{q}d)] = 0$$

*Research supported by NSERC, grant number A3828. This talk is based on the papers with Alakabha Datta and Harry J. Lipkin: - Phys. Lett. **B529** (2002) 93-98 [hep-ph/0111336], Phys.Lett.**B540** (2002) 97-103 [hep-ph/0202235] and Phys.Lett. **B544** (2002) 145-153 [hep-ph/0206155]. These papers contain details and references that could not be included here.

The following relations for amplitudes are tests of these selection rules:

$$\begin{aligned} A_L(B^0 \rightarrow J/\psi \rho^0) &= A_L(B^0 \rightarrow J/\psi \omega), \\ A_L(B_s \rightarrow J/\psi \rho^0) &= A_L(B_s \rightarrow J/\psi \omega) \\ &= A_L(B^0 \rightarrow J/\psi \phi) = 0 \end{aligned}$$

(L denotes any partial wave for the vector-vector final state)

All other decays are described by the two transitions:

$$\begin{aligned} B(\bar{b}q) &\rightarrow J/\psi \bar{d}q \rightarrow J/\psi M(\bar{d}q) \\ B(\bar{b}q) &\rightarrow J/\psi \bar{s}q \rightarrow J/\psi M(\bar{s}q) \end{aligned}$$

where the decay amplitudes are described as the product of a \bar{b} decay amplitude and a hadronization function for the combination of a quark-antiquark pair making the final meson. Charge conjugation symmetry implies no $SU(3)$ breaking in the final state interactions and

$$A(B_s \rightarrow J/\psi \bar{K}^{*0})_L = F_{CKM}^L \cdot A(B_d \rightarrow J/\psi K^{*0})_L$$

where F_{CKM}^L is a factor depending on the ratios of the CKM matrix elements and the ratio of various weak interaction diagrams contributing to B_d and B_s decays. For the (dominant) tree- and penguin-diagram contributions with a charmed quark loop, $F_{CKM}^L = V_{cd}/V_{cs}$.

Including $SU(3)$ symmetry gives an additional set of predictions

$$A_L(B_d^0 \rightarrow J/\psi \rho^0) = A_L(B_d^0 \rightarrow J/\psi \omega)$$

$$\begin{aligned}
&= A_L(B_s \rightarrow J/\psi \bar{K}^{*0})/\sqrt{2} \\
A_L(B_s \rightarrow J/\psi \phi) &= A_L(B^0 \rightarrow J/\psi K^{*0})
\end{aligned}$$

3. B decays into charmonium and a pseudoscalar meson

Decays involving η or the η' mesons in the final state have been unexplained by the standard treatments of these decays. Mixing, in general, involves four different radial wave functions and cannot be described by diagonalizing a simple 2×2 matrix with a single mixing angle. Write the normalized $\eta - \eta'$ wave functions as

$$\begin{aligned}
|\eta\rangle &= \cos \phi |N\rangle - \sin \phi |S\rangle \\
|\eta'\rangle &= \sin \phi' |N'\rangle + \cos \phi' |S'\rangle
\end{aligned}$$

where $|N\rangle$, $|N'\rangle$, $|S\rangle$ and $|S'\rangle$ are respectively arbitrary isoscalar non-strange and strange quark-antiquark wave functions. When, traditionally, the $\eta - \eta'$ mixing is described by a single mixing angle,

$$|N\rangle = |N'\rangle, \quad |S\rangle = |S'\rangle, \quad \phi = \phi',$$

and, including phase space effects, gives the experimentally measurable quantities,

$$r_d \equiv \frac{p_{\eta'}^3 \Gamma(\bar{B}^0 \rightarrow J/\psi \eta)}{p_{\eta}^3 \Gamma(\bar{B}^0 \rightarrow J/\psi \eta')} = \cot^2 \phi,$$

$$r_s \equiv \frac{p_{\eta'}^3 \Gamma(\bar{B}_s^0 \rightarrow J/\psi \eta)}{p_{\eta}^3 \Gamma(\bar{B}_s^0 \rightarrow J/\psi \eta')} = \tan^2 \phi,$$

with

$$r = \sqrt{r_d r_s} = 1.$$

Any large deviation of r from 1 would indicate evidence of non-standard $\eta - \eta'$ mixing.

If the pseudoscalar mesons have the same radial wave functions $SU(3)$ symmetry relates the amplitudes:

$$\begin{aligned}
A(B_d \rightarrow J/\psi N) &= A(B_s \rightarrow J/\psi \bar{K}^0)/\sqrt{2} \\
&= A(B_s \rightarrow J/\psi \pi^0), \\
A(B_s \rightarrow J/\psi S) &= A(B_d \rightarrow J/\psi K^0)
\end{aligned}$$

and gives sum rules for standard mixing that are independent of the mixing angle

$$\begin{aligned}
|A(B_d \rightarrow J/\psi \eta)|^2 + |A(B_d \rightarrow J/\psi \eta')|^2 \\
&= |A(B_s \rightarrow J/\psi \bar{K}^0)|^2/2 \\
|A(B_s \rightarrow J/\psi \eta)|^2 + |A(B_s \rightarrow J/\psi \eta')|^2 \\
&= |A(B_d \rightarrow J/\psi K^0)|^2.
\end{aligned}$$

Charge conjugation relates the ratios of B_d and B_s decays to $J/\psi \eta$ and $J/\psi \eta'$,

$$r_{\eta} = \frac{p_{B_s \eta}^3 \Gamma(B_d \rightarrow J/\psi \eta)}{p_{B_d \eta}^3 \Gamma(B_s \rightarrow J/\psi \eta)} = (F_{CKM})^2 \cdot \cot^2 \phi,$$

$$r'_{\eta} = \frac{p_{B_s \eta'}^3 \Gamma(\bar{B}^0 \rightarrow J/\psi \eta')}{p_{B_d \eta'}^3 \Gamma(\bar{B}_s \rightarrow J/\psi \eta')} = (F_{CKM})^2 \cdot \tan^2 \phi,$$

where $F_{CKM} = A(\bar{b} \rightarrow J/\psi \bar{d})/A(\bar{b} \rightarrow J/\psi \bar{s})$, and predicts

$$r_B = \sqrt{r_{\eta} r'_{\eta}} = (F_{CKM})^2.$$

We can also describe the *branching ratios for eight transitions in terms of three parameters* F_{CKM} , ϕ and an overall normalization.

If these relations hold experimentally, the standard mixing and the value of the mixing angle will be confirmed and established, the validity of $SU(3)$ symmetry for these transitions will be confirmed, and the value of F_{CKM} will determine the ratio of the penguin to tree contributions to the decay $B_d \rightarrow J/\psi K_S$ which is the “golden channel” for CP violation experiments. Otherwise, we get clues to new physics.

4. $\eta - \eta'$ mixing in semileptonic charmed meson decays

In addition to B decays we can also use $D(D_s) \rightarrow \eta(\eta') l \nu$ to get clean tests for mixing. Define the two ratios

$$\begin{aligned}
r_{\eta} &= \frac{\Gamma(D \rightarrow \eta l \nu)}{\Gamma(D_s \rightarrow \eta l \nu)} \\
r_{\eta'} &= \frac{\Gamma(D \rightarrow \eta' l \nu)}{\Gamma(D_s \rightarrow \eta' l \nu)}
\end{aligned}$$

In the U spin limit,

$$r_D = \sqrt{r_{\eta} r_{\eta'}} = 1$$

A large deviation of r_D from 1 would indicate evidence of non standard $\eta - \eta'$ mixing since such a deviation is unlikely to originate from U spin breaking.

Including q^2 phase space gives the two ratios

$$R_d(q^2) = \frac{p_{\eta'}^3 \frac{d\Gamma}{dq^2}(D_d \rightarrow \eta l \nu)}{p_{\eta^3} \frac{d\Gamma}{dq^2}(D_d \rightarrow \eta' l \nu)} = \cot^2 \phi,$$

$$R_s(q^2) = \frac{p_{\eta'}^3 \frac{d\Gamma}{dq^2}(D_s \rightarrow \eta l \nu)}{p_{\eta^3} \frac{d\Gamma}{dq^2}(D_s \rightarrow \eta' l \nu)} = \tan^2 \phi,$$

with standard mixing, and

$$R = R_d(q^2)R_s(q^2) = 1$$

for any value of q^2 . Again, a deviation of R from 1 or a q^2 dependence for R , R_d and R_s would indicate evidence of non-standard mixing.

5. Charmless B Decays to Final States with Radially Excited Vector Mesons

A meson in the final state of a nonleptonic decay could be in a radially excited state. Define the ratios

$$R_{\rho^+} = BR(\bar{B}^0 \rightarrow \pi^- \rho^{+'})/BR(\bar{B}^0 \rightarrow \pi^- \rho^+),$$

$$R_{\rho^0} = BR(\bar{B}^- \rightarrow \pi^- \rho^{0'})/BR(\bar{B}^- \rightarrow \pi^- \rho^0),$$

$$R_\omega = BR(\bar{B}^- \rightarrow \pi^- \omega')/BR(\bar{B}^- \rightarrow \pi^- \omega),$$

$$R_\phi = BR(\bar{B}^- \rightarrow \pi^- \phi')/BR(\bar{B}^- \rightarrow \pi^- \phi),$$

where ρ', ω' and ϕ' are radially excited states.

We need to diagonalize the mass matrix,

$$\begin{aligned} < q'_a \bar{q}'_b, n' | M | q_a \bar{q}_b, n > = \\ & (m_a + m_b + E_n) \\ & + \delta_{aa'} \delta_{bb'} \frac{B}{m_a m_b} \vec{s}_a \cdot \vec{s}_b \psi_n(0) \psi_{n'}(0), \end{aligned}$$

where $\vec{s}_{a,b}$ and $m_{a,b}$ are the quark spin operators and masses. Here $n = 0, 1, 2$ and the basis states for the isovector mesons are chosen as $|N, I = 1, I_3 = 1\rangle = -|u\bar{d}\rangle$, $|N, I = 1, I_3 = 0\rangle = |u\bar{u} - d\bar{d}\rangle/\sqrt{2}$ and $|N, I = 1, I_3 = -1\rangle = |d\bar{u}\rangle$. E_n is the excitation energy of the n^{th} radially excited state and B is the strength of the hyperfine

interaction. For $\omega - \phi$ mixing, the mass matrix has the additional flavor mixing term

$$+ \delta_{ab} \delta_{a'b'} \frac{A}{m_a m_b} \psi_n(0) \psi_{n'}(0).$$

(The basis states for the isoscalar mesons $|N >_n = |u\bar{u} + d\bar{d}\rangle_n/\sqrt{2}$ and $|S >_n = |s\bar{s}\rangle_n$ for the non-strange and strange wave functions). Different models have only a slight effect. Typically,

$$R_{\rho^+} : R_{\rho^0} : R_\omega : R_\phi = 2 : 2 : 2.5 : 6$$

There are two main conclusions:– *Although mixing between radially excited states and the ground state is small, decays to excited states are enhanced and, in particular, there is a large enhancement for R_ϕ .* (This decay is suppressed in the standard model)

6. Non Standard $\eta - \eta'$ mixing and Nonleptonic B and Λ_b Decays

Mixtures of the ground state and radially excited $q\bar{q}$ states can alter the high momentum behavior of the η and η' wave functions. The decays $B \rightarrow \eta(\eta') K(K^*)$ are dominated by the penguin diagrams since the tree term is color and CKM suppressed. With factorization we can have the kaon leaving the weak vertex with its full momentum and the remaining quark combining with the spectator quark to form the final $\eta(\eta')$ meson, or the \bar{s} quark in the QCD penguin combining with the s quark from the $b \rightarrow s$ transition to form the $\eta(\eta')$. Another possibility is one in which a $q\bar{q}$ pair (where $q = u, d, s$) appearing in the same current in the effective Hamiltonian, hadronizes to the $\eta(\eta')$. This is often referred to the OZI suppressed term. However, it is not as simple as this when the η' is in the final state because the OZI suppressed terms add constructively. We expect the OZI suppressed terms to be more important in $B \rightarrow KP$ than in $B \rightarrow KV$ decays since in J/ψ and Υ decays we know that the OZI-forbidden process requires three gluons for coupling to a vector meson and two gluons for coupling to a pseudoscalar. Thus the OZI suppressed terms should have a smaller contribution to the $B \rightarrow K\rho^0(\omega)$ and $B \rightarrow K\phi$ decays than to $B \rightarrow K\eta$ and $B \rightarrow K\eta'$ decays. In the absence of

a model independent extraction of ϕ we will use the Isgur mixing, $\phi = 45^\circ$, as our standard.

$$\begin{aligned} |\eta\rangle_{std} &= \frac{1}{\sqrt{2}} [N_0 - S_0], \\ |\eta'\rangle_{std} &= \frac{1}{\sqrt{2}} [N_0 + S_0]. \end{aligned}$$

For the $\eta - \eta'$ system, including radial excitations, we diagonalize the mass matrix as in the $\omega - \phi$ case. The value of the parameter A is significantly changed. Nonet symmetry is broken; members of the pseudoscalar nonet do not all have the same radial wave function. This non-standard mixing gives important differences for the nonleptonic decays $B \rightarrow \eta(\eta')K(K^*)$.

$$\begin{aligned} |\eta\rangle_g &= 0.85 |\eta\rangle_{std} + 0.23 |\eta'\rangle_{std}, \\ |\eta'\rangle_g &= 0.91 |\eta'\rangle_{std} - 0.025 |\eta\rangle_{std}. \end{aligned}$$

For $B \rightarrow \eta(\eta')K(K^*)$ we find that they are *dominated by the penguin diagrams* and that the *tree term is color and CKM suppressed*. The ratio r can be as small as 0.2. In the absence of OZI terms there are definite predictions about the branching ratios $B \rightarrow \eta K/B \rightarrow \eta' K$ and $B \rightarrow \eta K^*/B \rightarrow \eta' K^*$. A parity selection rule for the decays $B \rightarrow \eta(\eta')K^{(*)}$ fixes the relative phase between the amplitudes for the strange and non-strange penguins causing interference.

$$\begin{aligned} R_K &\approx \left| \frac{f_K F_\eta^+ + f_\eta^s F_K^+}{f_K F_{\eta'}^+ + f_{\eta'}^s F_K^+} \right|^2 \sim 0.03, \\ R_{K^*} &\approx \left| \frac{f_{K^*} F_\eta^+ - f_\eta^s F_{K^*}^+}{f_{K^*} F_{\eta'}^+ - f_{\eta'}^s F_{K^*}^+} \right|^2 \sim \frac{1}{R_K} = 33, \end{aligned}$$

the last from neglecting form factor differences. The small value for R_K is consistent with the current experimental limits.

For Λ_b only one diagram contributes:

$$R_\Lambda \approx \left| \frac{f_\eta^s F_\Lambda}{f_{\eta'}^s F_\Lambda} \right|^2 \sim 1$$

Additional sum rules connect B decays to $K\eta(\eta')$ and $K\pi$ final states. One such sum rule is,

$$R = \frac{\Gamma[B^\pm \rightarrow K^\pm \eta'] + \Gamma[B^\pm \rightarrow K^\pm \eta]}{\Gamma[B^\pm \rightarrow K^\pm \pi^0]} \leq 3$$

Standard mixing gives $R \approx 3$. Non-standard $\eta - \eta'$ mixing gives $R \approx 6$, *consistent with experiment*. Similarly, for K^* final states,

$$\begin{aligned} \frac{\Gamma[B^\pm \rightarrow K^{*\pm} \eta]}{\Gamma[B^\pm \rightarrow K^{*\pm} \pi^0]} &\approx |\sqrt{2} + 1|^2 \approx 6, \\ \frac{\Gamma[B^\pm \rightarrow K^{*\pm} \eta']}{\Gamma[B^\pm \rightarrow K^{*\pm} \pi^0]} &\approx |\sqrt{2} - 1|^2 \approx \frac{1}{6}. \end{aligned}$$

New radial admixtures in the wave functions of the η and η' can increase the $B \rightarrow K\eta'$ decay but not the $B \rightarrow K^*\eta'$ decay, as in other OZI-violating models. Thus there is a *suppression* for $B \rightarrow K^*\eta'$ decay, an *enhancement* for $B \rightarrow K^*\eta$ decay relative to $B \rightarrow K^*\pi$ and an *enhancement* for $B \rightarrow K\eta'$ decay relative to $B \rightarrow K\pi$.

For the OZI suppressed contribution,

$$\frac{A_{\eta' K(K^*)}^{OZI}}{A_{\eta K(K^*)}^{OZI}} \approx \frac{(1 + \frac{1}{\sqrt{2}})}{(1 - \frac{1}{\sqrt{2}})} \sim \frac{A_{\eta' \Lambda}^{OZI}}{A_{\eta \Lambda}^{OZI}} \sim 6$$

A complete calculation gives

$$R_K = 0.035, R_{K^*} = 5.6, R_\Lambda = 0.4.$$

in reasonable agreement with experiment.

7. Summary and Conclusions

Two body decays of $B_d \rightarrow J/\psi M$ and $B_s \rightarrow J/\psi M$ have interesting relations among decay amplitudes. We propose tests for the standard $\eta - \eta'$ mixing. $SU(3)$ symmetry gives additional relations among the eight CP eigenstates $J/\psi K_S$, $J/\psi \eta$, $J/\psi \eta'$ and $J/\psi \pi^0$ and the value of the $\eta - \eta'$ mixing angle. Semileptonic D decays can also give important information about the standard $\eta - \eta'$ mixing. Weak decays of a B meson to final S-wave radially excited give ratios of nonleptonic decays $B \rightarrow \rho' \pi/B \rightarrow \rho \pi$, $B \rightarrow \omega' \pi/B \rightarrow \omega \pi$ and $B \rightarrow \phi' \pi/B \rightarrow \phi \pi$ transitions to the excited states comparable, or enhanced relative, to ground state transitions. Radial mixing in the $\eta - \eta'$ system leads to appreciable effects that gives a good description of the existing data on $B \rightarrow \eta(\eta')$. There are predictions for as yet unobserved decays unique to our model. Finally another unique prediction of our model is that there is a modest enhancement of $\Lambda_b \rightarrow \Lambda \eta'$ relative to $\Lambda_b \rightarrow \Lambda \eta$, unlike in the B system.